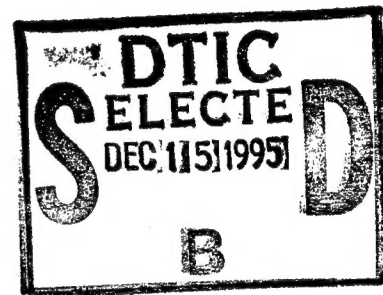


**US Army Corps  
of Engineers**  
Waterways Experiment  
Station

Technical Report CERC-95-18  
October 1995

# Compilation of Suspended-Load Point-Transport Theories

by *Thomas G. Drake, Evans-Hamilton, Inc.*  
*Thomas E. White, WES*



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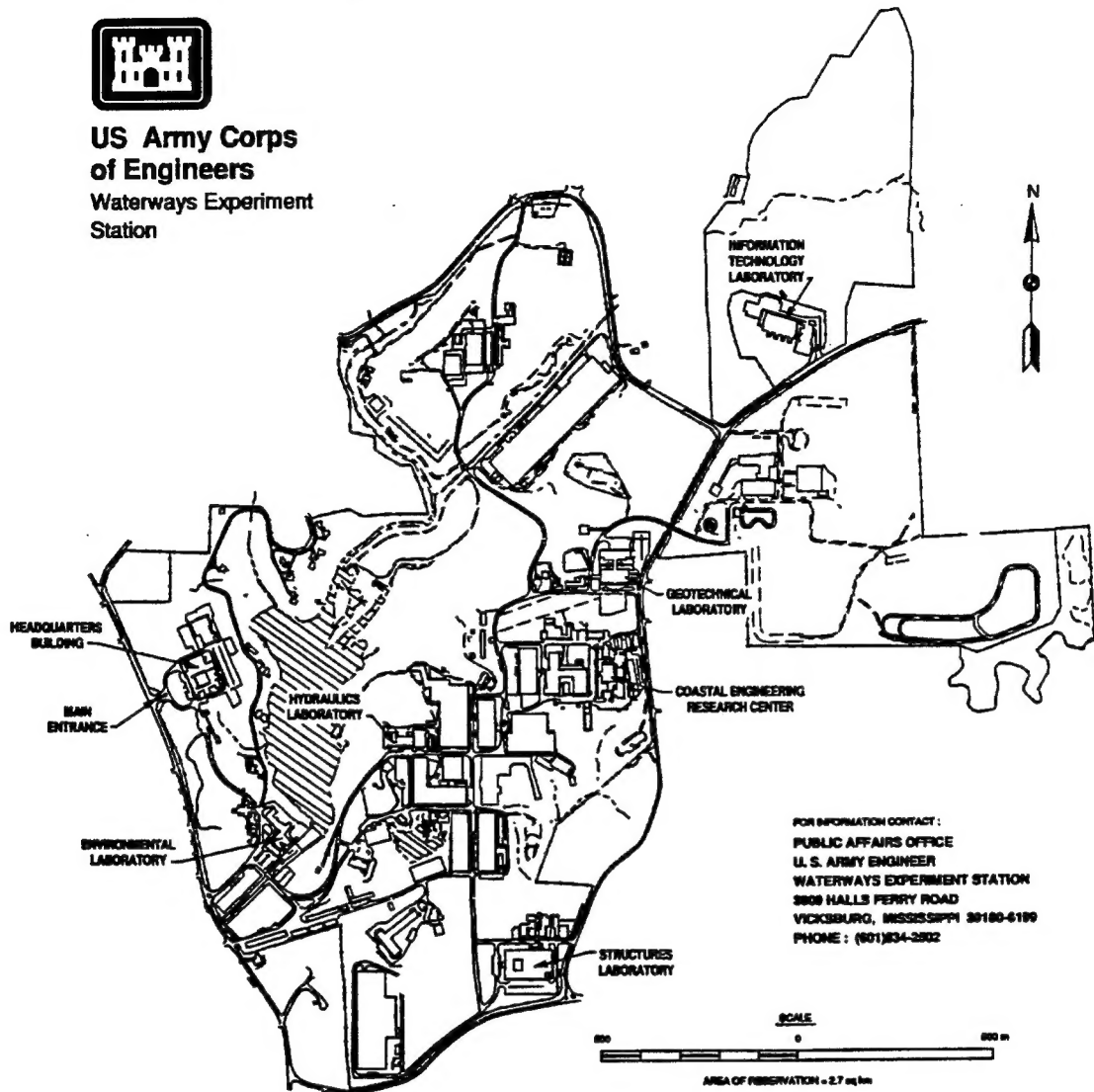
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# Preface

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This contract report summarizes a literature review of suspended-sediment transport theories under oscillatory flow conditions performed by Dr. Thomas G. Drake. The work was performed under Evans-Hamilton, Inc., Contract No. DACW39-92-D-0012 with the U.S. Army Engineer Waterways Experiment Station's (WES) Coastal Engineering Research Center (CERC).

This report supplies a list of suspended-sediment transport theories to be tested with field data collected by personnel in the Prototype Measurement and Analysis Branch, CERC. Dr. Thomas White was Principal Investigator and provided technical oversight for Work Unit 32685 "Field Tests of Sediment Transport Theories" in the Coastal Research Program at CERC. The work unit continues under the Inlets Research Program.

WES supervisors during this report's preparation were Branch Chief William L. Preslan, Division Chief Thomas W. Richardson, Program Manager Carolyn M. Holmes, Assistant Director of CERC Charles C. Calhoun, Jr., Director of CERC Dr. James R. Houston, Director of WES Dr. Robert W. Whalin, and Commander of WES COL Bruce K. Howard, EN.

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# 1 Introduction

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## Background

Numerous theories are available to predict the transport of cohesionless sediments by suspension in the surf zone. The purpose of this report is to survey the available literature for such theories; and, subject to certain criteria discussed below, to differentiate between unique theories and essentially redundant ones. For each of the unique theories, a common set of variables is used to formulate an explicit mathematical expression for the suspended sediment transport rate. Redundant theories are those which differ only by numerical coefficients from any of the unique theories, having otherwise the same functional form. Accompanying each unique formula is an outline of its derivation, and a list of the assumptions underlying the derivation. The selected formulae will form a basis for testing theoretical predictions with field measurements of suspended-sediment transport and requisite fluid and sediment quantities at a point within the surf zone or immediately adjacent to it.

## Organization

The organization of the report is as follows: first, the criteria used to select theories from the literature are presented; second, a section on theory discusses general considerations, and presents each of the unique formulas, their derivation, and assumptions. A table of selected theories summarizes formulae for testing field measurements. An extensive bibliography is presented in addition to the section describing references cited in the body of the report. Finally, Appendix A describes the notation used in this report.

## Selection Criteria

This report emphasizes theories for suspended sediment transport in or immediately adjacent to the surf zone in combined wave-current nearshore flows under conditions such that quasi-unidirectional currents do not dominate the combined flow.

The criteria used to select theories for further consideration are as follows:

- a. Theories must be valid within the surf zone.
- b. Theories must specifically predict suspended-sediment transport at a point; theories yielding only predictions of transport averaged over any spatial coordinate other than the vertical are excluded. Acceptable theories may thus require as input depth-averaged fluid and sediment quantities, but theories predicting only transport averaged over the width of the surf zone, for example, are excluded.
- c. Specifically excluded are formulations using only bed elevation measurements as input and the kinematic constraints of the sediment conservation equation to calculate spatial gradients in the sediment transport rate. Such formulations do not provide predictions of the sediment transport rate based on measured fluid-motion and grain-size parameters.

The comprehensive literature search focused primarily on theories presented in refereed journals and government reports, because they are readily available in most libraries and have been peer reviewed. However, a considerable sediment-transport literature exists outside this group, primarily in the form of conference proceedings, and these reports are included in the survey to the extent that such reports are available. Fortunately, there is considerable overlap between the refereed and so-called "gray" literature, so that essentially all useful transport formulae published in the English language (or translated to English from the original literature) are included here.



## 2 Theory

---

Theories for transport of suspended sediment fall into three broad categories: those based on explicit energy arguments; those based on the diffusion theory or modifications to it; and quasi-empirical predictors of sediment transport. With few exceptions, theories for suspended sediment transport in the surf zone are directly based on or are fundamentally related to theories first developed to describe steady unidirectional flows in fixed-bed channels. Thus, the original theories for unidirectional flow are occasionally referenced to clarify the derivations of surf zone transport theories.

Because the so-called "energetics" models require the measurement of far fewer quantities to predict transport in the surf zone, they are greatly favored in nearshore field studies. The following sub-sections outline the energy-based approach of Bagnold (1963; 1966), critically examine assumptions made in a number of energetics-based theories that extend Bagnold's approach, and list the limitations of those theories; then a similar approach is taken with the diffusion theories. Finally, two commonly used quasi-empirical predictors of sediment transport are critically examined. Particular emphasis is given to calibration constants and free parameters in the theories.

### **Energetics-Based Theories for Suspended-Sediment Transport**

Energetics-based theories for suspended sediment transport following the assumptions of Bagnold (1963; 1966) have been proposed by numerous investigators. The form of the general theory and its assumptions are presented below, followed by a critical examination of the modifications to it for unsteady oscillatory flow proposed by Bowen (1980), Bailard (1981; 1982; 1987) and Roelvink and Stive (1989).

Bagnold (1966) derived relations for the transport of sediment as bed load and suspended load in unidirectional streams based on the assumption that a fixed fraction of the total stream power is expended moving bed load, and that a fixed fraction of the remaining available power is expended transporting suspended sediment. Bagnold (1966, p. I-4) defined bed load as "that part of

the load which is supported wholly by a solid-transmitted stress," and suspended load as "that part which is supported by a fluid-transmitted stress." Taking the total stream power  $\omega$  as

$$\omega = \rho gh S \bar{u} \quad (1)$$

where

$\rho$  = fluid density  
 $g$  = gravitational acceleration  
 $h$  = flow depth  
 $S$  = energy slope  
 $\bar{u}$  = mean flow velocity

Bagnold assumed that the immersed-weight transport rate of suspended load is

$$i_s = \frac{e_s \bar{u} (1 - e_b)}{W} \omega \quad (2)$$

where

$e_s$  = suspended-load transport efficiencies  
 $e_b$  = bed load  
 $W$  = sediment fall velocity<sup>1</sup>

Unfortunately, there is an error in Bagnold's derivation of the suspended sediment efficiency  $e_s$ , that appears to seriously compromise the theory. In Bagnold's Equation 14 (p. I-14), he mistakenly suggests that the momentum flux due to downward-directed momentum moving downward is opposite that due to upward-directed momentum moving upward, when in fact both have the same sign.

Despite its deficiencies, Bagnold and others have extended the energetics approach to predict sediment transport in unsteady oscillatory flows. For oscillatory flows, Bagnold (1963) proposed a similar relation for the total load:

$$i_{\theta} = K_B \omega \frac{u_{\theta}}{u_m} \quad (3)$$

---

<sup>1</sup> For convenience, symbols are listed in the notation (Appendix A).

where

$K_B$  = constant

$u_\theta$  = current velocity in the  $\theta$  direction

$u_m$  = maximum wave orbital velocity measured at the same distance above the bed as  $u_\theta$

$i_\theta$  = transport in the  $\theta$  direction

Although not considered further here, functionally similar expressions for the total load have been developed by many investigators. The following paragraphs outline and contrast the modifications to Bagnold's ideas proposed by Bowen (1980), Bailard (1981; 1982; 1987), and Roelvink and Stive (1989).

Common to each of the energetics-based theories are the following assumptions:

- a. The immersed sediment transport rate  $i$  is proportional to  $\omega$ , the local rate of energy dissipation. Assuming that the shear stress  $\tau$  is quadratic in the velocity  $u$

$$\tau = \rho c_f u^2 \quad (4)$$

where  $c_f$  is the bed drag coefficient, and the rate of energy dissipation  $\omega$  is assumed to be

$$\omega = \rho c_f |u|^3 \quad (5)$$

- b. Equations 4 and 5 assume zero phase difference between stress and velocity.
- c. The bathymetry is free of bed forms, and has parallel contours.
- d. Flow accelerations play no role in the sediment transport processes.
- e. The sediment grain-size distribution is completely characterized by a single constant fall velocity  $W$ , which is independent of concentration.
- f. There is no provision for initiation of sediment motion.
- g. Sediment transport responds instantaneously to changes in flow conditions.
- h. An implicit assumption is that the concentration of suspended sediment is sufficiently low that stratification effects can be neglected (e.g., Villaret and Trowbridge (1991)).

- i. Vertical fluid velocities are assumed to play no role in the depth-integrated transport process, beyond their obvious role in suspending sediment. There is mounting evidence (e.g. Bowen (1990), summary in Nielsen (1992) that vertical velocities can be important, particularly if sediment concentration decreases slowly with distance from the bed.

Generalizing the expression developed by Bailard (1981), the time-averaged immersed suspended-sediment transport rate at a point is

$$\langle \vec{\tau} \rangle = \rho c_f \frac{e_s}{W} \left[ \langle |\vec{u}|^3 \vec{u} \rangle + K_1 + K_2 \langle |\vec{u}|^5 \rangle \hat{i} \right] \quad (6)$$

where angle brackets indicate time averaging,  $\vec{u}$  is the instantaneous fluid velocity vector,  $\hat{i}$  is the unit vector in the downslope direction, and the constants  $K_1$  and  $K_2$  are discussed below. For purposes of discussing proposed variations of Equation 6, the primitive equation (e.g., Guza and Thornton (1985)) for suspended-sediment transport in the cross-shore ( $x$ ) direction in the case  $K_1=0$  is

$$\langle i \rangle = \frac{e_s c_f \rho u^3 |u|}{W(1-K_2 u)} \quad (7)$$

Here  $u$  is the velocity component parallel to the local bed slope.

### Bowen (1980)

Bowen adopted Bagnold's model as a simple starting point for developing models of onshore-offshore transport. Explicitly supposing the maximum orbital wave velocity  $u_m$  to dominate the total velocity

$$u = u_m \cos(\sigma t) + U_1 \quad (8)$$

where  $\sigma$  is the radial wave frequency, Bowen expanded Equation 8 and investigated several cases for various forms of the perturbation  $U_1$ .

Both Bagnold and Bowen derived  $K_1=0$  and  $K_2=\beta/W$  (implicitly assuming  $\beta \ll 1$ , so that  $\tan \beta \approx \beta$ , where  $\beta$  = angle of bed slope). Bowen specifies temporal averaging over many wave periods.

### Bailard (1981; 1982; 1987)

Bailard derived a slightly different form of Equation 6, keeping only the first two terms in the binomial expansion of Equation 7. Bailard's derivation gives  $K_1=0$  and  $K_2 = e_s \tan \beta / W$ . As Bailard (1981, p. 10,952) notes, "this

difference arises because Bagnold assumed that the stream power contribution from the suspended sediment load contributes directly to the suspended sediment transport rate instead of through an efficiency factor..."

The possibility of "autosuspension" arises when the factor  $(1-K_2u)$  in the denominator of Equation 7 approaches zero. The expansion leading to (6) is singular in this case. First noted by Knapp (1938), autosuspension refers to turbulent flows transporting suspended sediment in which the rate of energy input to the flow by the downslope motion of the sediment equals or exceeds the rate at which energy is expended by the flow to suspend the particles. In such cases, additional sediment can be suspended by the flow with no net energy input, and thus the concentration is limited only by sediment availability.

The models of Bailard and Bowen both admit the possibility of autosuspension, though the quantitative criteria differ. Bailard (1981) and Pantin (1979) predict autosuspension when

$$\tan\beta \geq \frac{W}{e_s u} \quad (9)$$

and Bagnold (1962) and Bowen (1980) predict autosuspension when

$$\tan\beta \geq \frac{W}{u} \quad (10)$$

Clearly such conditions can arise in the surf zone, and the viability of the autosuspension concept has been debated in the literature. Critical laboratory experiments by Southard and Mackintosh (1981) show that the Bagnold autosuspension criteria (Equation 10) is not valid. Their experiments, which do not test the more conservative criterion (Equation 9) proposed by Bailard and Pantin, strongly suggest that the energy balance in these flows is not merely a matter of comparing the energy input by the downslope motion of suspended particles and the energy expended to suspend the particles. That comparison ignores the changes induced by the presence of suspended particles on the turbulent structure of the flow; and perhaps of greater importance, does not include the effects of high suspended-sediment concentration on the boundary resistance. Turbulence is damped by density stratification at high concentrations, thus limiting the vertical transport of mass and momentum (McLean 1991). Thus boundary resistance can be significantly decreased as the suspended load increases, in turn reducing the ability of the flow to entrain more sediment into suspension. It is this balance between entrainment and deposition at the bed, Southard and Mackintosh argue, that governs the equilibrium concentration of sediment in the flow. Further discussion of the issues can be found in Parker (1982), Pantin (1982), Paola and Southard (1983), and Seymour (1986).

### Roelvink and Stive (1989)

The theory proposed by Roelvink and Stive (RS) assumes  $K_2 = 0$ , and derives  $K_1$  based on a one-equation model for turbulence produced by breaking waves. The overall goal of their theory is to incorporate models for a variety of cross-shore flows, such as asymmetric oscillatory flow, wave grouping-induced long-wave flow, breaking-induced turbulent flow, and momentum decay-induced undertow. The resulting model for the fluid flow, derived for cross-shore transport only, can be simplified and used for comparison with field data. In their model, the parameter  $K_1$  is

$$K_1 = \beta_d \{ k_t [\exp(h/H_{rms}) - 1]^{-1} \}^{3/2} \quad (11)$$

where  $\beta_d$  is a dimensionless coefficient (assumed to be  $\approx 1$ ),  $H_{rms}$  = root-mean-square wave height, and  $k_t$  is the turbulent kinetic energy per unit mass

$$k_t = \frac{1}{2} [\langle u'^2 \rangle + \langle v'^2 \rangle + \langle w'^2 \rangle] \quad (12)$$

where primes indicate fluctuating velocity components. Although the RS model calculates  $k_t$  by an iterative solution of a system of equations, available field data may be used to determine  $k_t$  directly. Alternatively, the average dissipation  $\epsilon_d$  per unit mass due to wave breaking might be approximated following Thornton and Guza (1986):

$$\epsilon_d = \frac{3}{16} \sqrt{\pi} g \frac{B^3}{\gamma^4 h^5} f_p H_{rms}^7 \quad (13)$$

where  $f_p$  is the peak frequency,  $\gamma$  is an empirically determined ratio of root-mean-square wave height  $H_{rms}$  to the water depth  $h$  in the surf zone (Thornton and Guza 1983), and is typically about  $0.4 \pm 0.1$ . The coefficient  $B$  accounts for differences in wave breaking; it is less than one for spilling breakers and near unity for plunging breakers (Thornton and Guza 1986). Then, assuming a balance between the dissipation  $\epsilon_d$  and the creation of turbulent kinetic energy gives

$$k_t^{3/2} = \epsilon_d \quad (14)$$

which supplies all the information necessary to determine  $K_1$ .

### Humiston (1993)

Following Guza and Thornton (1985), Humiston modeled contributions of infragravity and incident waves and quasi-steady currents to cross-shore transport of both bed load and suspended load. The total velocity is decomposed into quasi-steady and oscillatory components

$$\vec{u}_t = (\tilde{u} + \bar{u}) \hat{i} + (\tilde{v} + \bar{v}) \hat{j} \quad (15)$$

where overbars indicate the quasi-steady quantities, tildes indicate oscillatory ones, and  $\hat{i}$  and  $\hat{j}$  are unit vectors in the on-offshore and alongshore directions.

The oscillatory terms are further expanded into infragravity  $\tilde{u}_i$  ( $f < 0.05$  Hz) and incident  $\tilde{u}_s$  ( $f > 0.05$ ) components such that

$$\tilde{u} = \tilde{u}_i + \tilde{u}_s \quad (16)$$

and

$$\tilde{v} = \tilde{v}_i + \tilde{v}_s \quad (17)$$

The magnitude of the total velocity is then

$$|\vec{u}_t| = [ \tilde{u}^2 + \tilde{v}^2 + \bar{u}^2 + \bar{v}^2 + 2(\tilde{u}\bar{u} + \tilde{v}\bar{v}) ]^{1/2} \quad (18)$$

This decomposition of the oscillatory flow velocities may be useful in comparing model outputs with field measurements.

### Bed friction and suspended sediment efficiency factors

The bed friction and suspended sediment efficiency factors are two important parameters in the energetics-based theories. Although both are assumed constant in the theories presented above, it is likely that the bed friction factor is a complex function of fluid-motion parameters, in particular, the properties of the bed-load layer and any small-scale bed roughness generated by the bed-load process. The choice of suspended sediment efficiency factor lacks theoretical guidance to indicate its value, and must be determined by calibration. The following discussion focuses on Bailard's studies, which typically form the basis for the choice of constants in other investigations.

Bailard (1981) determined the suspended sediment efficiency factor using selected data from laboratory and field experiments. He determined  $e_s = 0.025$  (as well as the bed-load efficiency factor, the bed friction factor, and a number of other parameters) from measured estimates of total longshore transport and velocity using an iterative scheme from Ostendorf and Madsen (1979). Estimated values of the parameters are not independent, and error analysis is difficult. Bailard (1987, p. 339) used  $e_s = 0.035$  for field data from Leadbetter and Torrey Pines Beaches as part of the nearshore sediment transport study. Other investigators have adopted Bagnold's (1966) estimate  $e_s = 0.015$  for unidirectional stream flow. Since the transport rate is directly proportional to  $e_s$ , errors of 100 percent or more may be expected in uncalibrated application of Equation 6 to field measurements due to errors in estimating  $e_s$  alone.

The measured values of  $c_f$  in the experiments Bailard (1981) used to determine  $e_s$ , range from 0.013 to 0.017 for the laboratory experiments; for the three Silver Strand field experiments (see Komar (1969) for original data) used in the analysis,  $c_f$  ranged from 0.0026 to 0.014. It should be noted that the values for  $c_f$  presented in Bailard's Table 2 (p. 10,951) are inconsistent with his Equations 42 and 43 (p. 10,946) using  $\Gamma = 0.13$  and values for other parameters from his Table 2. There is no discussion of the value adopted for  $c_f$  in the calculations in the paper, although subsequent papers (Bailard (1982), p. 1424) cite a value of  $c_f = 0.005$  based on Komar's (1969) Silver Strand data.

Because the bed friction factor  $c_f$  is an integral part of many longshore current models, there has been considerably more study of  $c_f$  than  $e_s$ . Shemdin et al. (1978) compiled values of  $c_f$  for various field studies of wave dissipation, and found a range of 0.005 to 0.3. Thornton and Guza (1986) determined  $c_f$  using different longshore models; for a linear bed shear stress model (Equation 14, p. 1167) they found the optimal  $c_f = 0.009 \pm 10.001$ , while for the nonlinear model (Equations 22 and 23, p. 1168) the best-fit  $c_f = 0.006 \pm 10.0007$ , significantly less than for the linear model. Their linear model corresponds to the quadratic shear stress model Equation 4 assumed in the energetics-based transport models. Thus, uncalibrated application of Equation 6 and its variants may introduce uncertainties of 100 percent and more into transport estimates due to errors in  $c_f$ .

## Diffusion-Based Theories for Suspended-Sediment Transport

Several theoretical approaches to suspended sediment transport seek to calculate transport based on a detailed picture of near-bed fluid motion, in particular, the computation of the boundary shear stress and the assumption that fluid turbulence diffuses sediment according to a Fickian diffusion law. The assumption of Fickian diffusion is not the only one possible, but alternatives to it (e.g., McTigue (1983)) have not been used in the context of nearshore transport and thus are not considered further here.

Rouse (1937) derived a diffusion-based theory for suspended sediment transport in steady, uniform unidirectional flow, which forms the basis for theories describing transport in unsteady, combined flows in the nearshore. The governing convection-diffusion equation for the volume concentration of sediment  $N$  is

$$\frac{\partial N}{\partial t} = W \frac{\partial N}{\partial z} + \frac{\partial}{\partial z} \left[ v_s \frac{\partial N}{\partial z} \right] \quad (19)$$



where  $\nu_s$  is the sediment diffusivity, often equated to the fluid eddy viscosity, and diffusion and advection in the  $x$  and  $y$  directions have been neglected. For steady uniform flow in a channel, the vertical distribution of suspended sediment is then

$$N = N_0 \left[ \frac{h-z}{z} \frac{z_0}{h-z_0} \right]^R \quad (20)$$

The Rouse number  $R$  is

$$R = \frac{W}{c_r \kappa u_*} \quad (21)$$

where  $c_r$  is an empirically determined constant of proportionality between the sediment diffusivity and the fluid eddy viscosity (often assumed to be unity),  $\kappa$  is von Karman's constant, and  $u_* = (\tau_b / \rho)^{1/2}$  is the friction velocity. The derivation of Equation 19 assumes a law-of-the-wall velocity profile; application of it requires specification of a boundary condition at  $z = z_0$ . Typical extensions to the Rouse formulation for unsteady flows include a time-varying and/or vertically varying eddy viscosity, and a bed-shear stress relation for the reference concentration  $N_0$ .

Kennedy and Locher (1972) and Smith (1977) present thorough discussions of the diffusion-based theories and their application to both oscillatory and unidirectional flows. Grant and Madsen (1979) use an eddy-diffusivity closure scheme to couple equations for conservation of fluid momentum and sediment mass to describe combined wave-current flows on the continental shelf. They solve the model equations using an iterative numerical technique. Glenn and Grant (1987) extended the work of Grant and Madsen (1979) to include stratification effects. Wikramanayake and Madsen (1992) used new relations for the reference concentration at the bed in an otherwise similar approach to suspended sediment transport under unbroken waves.

Diffusion-based theories have been extended to treat several special cases of sediment transport in the surf zone, but there have been no theories developed to treat the general case of combined wave-current flow that have been reduced to a single formula for comparison with field measurements; the analyses typically result in systems of coupled nonlinear equations that must be solved numerically. Major difficulties in applying such theories to field data include specification of initial and boundary conditions with sufficient detail to ensure unique solutions to the equations. A brief review of diffusion-based models for suspended sediment transport in the surf zone is presented below; however, none of the theories meet the selection criteria and thus are not included in Table 1.

In general, the instantaneous rate of suspended sediment transport is given by

$$\int \bar{u} N dz \quad (22)$$

where  $\bar{u}$  is the instantaneous fluid velocity, and the integral is taken from the bed to the free surface. Theories presented below use various approaches to evaluate the vertical distributions of the diffusivity  $v$ , in Equation 19 and  $\bar{u}$  in Equation 22, and to decompose  $\bar{u}$  into mean and fluctuating components due to currents, waves, and turbulence. Implicit in all the following formulas, unless otherwise noted, are the following assumptions:

- a. The horizontal velocity of the sediment particles is equal to the horizontal fluid velocity.
- b. Flow accelerations play no explicit role in the sediment transport processes.
- c. The concentration of suspended sediment is sufficiently low that stratification effects can be neglected.

As can be seen, assumptions for the diffusion-based theories are far fewer than for the energetics-based models, and means to rationally extend the theories are typically available, although often analytically intractable.

### **Dally and Dean (1984)**

The two-layer model proposed by Dally and Dean for cross-shore suspended sediment transport assumes that sediment motion occurs by two mechanisms. In the near-bottom layer, each grain is transported a small distance before redepositing on the bed. Derivation of this "first-order contribution," in their terminology, assumes linear wave theory to predict the mean horizontal distance traveled by a grain in one wave period. The second transport mechanism is a mean flow (derived from radiation stress and conservation of mass requirements) that operates in both the near-bed and upper layers. The sum of these sediment velocity contributions is multiplied by the sediment concentration profile to determine the vertical profile of suspended sediment transport using a finite-difference scheme. The concentration profile is modified from Rouse (1937) to include a contribution to the shear velocity due to wave-breaking-induced turbulence. Several parameters in the theory are not specified by Dally and Dean, and thus quantitative comparison of their theory with field measurements is not possible.

### **Deigaard, Fredsoe, and Hedegaard (1986)**

Using the analogy of a turbulent bore to describe a spilling breaker, Deigaard, Fredsoe, and Hedegaard (1986) solve numerically a one-equation model describing the transport and diffusion of turbulent kinetic energy in the

surf zone. The total eddy viscosity comprises contributions due to turbulence and to the production of energy in the combined wave-current motion. The latter contribution is calculated according to the theory of Fredsoe, Anderson, and Silberg (1985). Finally, the suspended sediment transport is calculated according to Engelund and Fredsoe (1976).

## **Semi-Empirical Models for Suspended Sediment Transport**

Given the difficulty in predicting suspended sediment transport in the nearshore from first principles, several engineering-oriented approaches have made extensive use of field data from evolving beaches and from large-scale laboratory studies to parameterize transport processes. The semi-empirical or schematic theories described below are based on the assumption that an equilibrium profile exists for any constant wave climate. Although that assumption is currently the subject of vigorous study, the schematic approaches are a practical, albeit crude, means of predicting sediment transport, particularly when detailed fluid-motion data are lacking and some a priori knowledge of the typical beach profile is available.

### **Larson and Kraus (1989)**

Founded on considerable field and laboratory data, the SBEACH model developed by Larson and Kraus (1989) (hereafter referred to as "LK") extends the energy dissipation models proposed by Kriebel and Dean (1985) and Kriebel (1986) to predict total net sediment transport in the surf zone and foreshore. These models are designed to make predictions about "large-scale profile features on intervals of tens of minutes" (LK, p.139). Thus they differ fundamentally in purpose from theories described above, which predict transport on a wave-by-wave basis. These models make the following assumptions:

- a. Beach profile change, and thus total net sediment transport, is mainly governed by breaking of short-period waves having periods in the approximate range of 3-20 s (LK, p.12). Thus infragravity waves are explicitly assumed to have little effect on sediment transport and resulting profile evolution.
- b. An equilibrium profile exists such that the energy dissipation per unit volume is uniform across the surf zone (Dean 1977). Dean's development of this hypothesis assumes linear wave theory, a constant ratio of wave height to water depth within the surf zone, and linear and parallel bottom contours to arrive at a characteristic equilibrium profile shape

$$h = Ax^{\frac{2}{3}} \quad (23)$$

where  $A$ , the profile shape parameter, is a function of particle size or fall velocity and  $x$  is distance offshore. Departures from this equilibrium profile provide the driving mechanism for sediment transport.

- c. Waves are normally incident, and only cross-shore transport is predicted.
- d. The direction of transport (i.e. onshore or offshore) is determined by an empirical criterion that differentiates between bar growth (offshore transport), and berm growth (onshore transport).
- e. The magnitude of the total net transport at a point is directly proportional to the difference between the energy dissipation rate per unit volume estimated from field measurements and the equilibrium dissipation rate determined according to Dean's (1977) hypothesis. The transport formula proposed by LK (p. 167) is

$$\langle i \rangle = \text{sgn} \left\{ M \left[ \frac{\langle H_0 \rangle}{WT} \right]^3 - \frac{2\pi \langle H_0 \rangle}{gT^2} \right\} (\rho_s - \rho) gK \left[ E - E_{eq} + \frac{\epsilon_\beta}{K} \frac{dh}{dx} \right] \quad (24)$$

when

$$E > E_{eq} - \frac{\epsilon_\beta}{K} \frac{dh}{dx} \quad (25)$$

and  $\langle i \rangle = 0$  otherwise. The sign or direction of transport (i.e.,  $\text{sgn}$ ) is determined by the quantity in curly brackets, based on an empirical criterion that differentiates between bar growth (offshore transport in the direction of increasing coordinate  $x$ ) and berm growth (onshore transport). The quantity  $dh/dx$  is the bed slope,  $H_0$  is the deepwater wave height,  $W$  is the sediment fall velocity,  $T$  is the wave period determined from the spectral frequency peak  $f_p$ ,  $M=0.0007$  is an empirical constant and  $\rho_s$  is sediment density. The magnitude of transport is determined from the quantity in the right-most square brackets, in which  $E$  is the energy dissipation rate per unit volume calculated according to assumptions discussed below,  $E_{eq}$  is the equilibrium energy dissipation rate, and  $\epsilon_\beta$  is a constant. The term involving the beach slope  $\epsilon_\beta h/dx$  is incorporated primarily to improve the numerical stability of the model (LK, p. 168). The value of the transport rate coefficient  $K$  is determined from experimental and field measurements, and is discussed in more detail below. The erosion-only model of Kriebel and Dean (1985) and Kriebel (1986) is a special case of Equation 24 in which transport is assumed to be directed offshore only, and the term involving the beach slope  $dh/dx$  is neglected.

The general form of Equation 24 implies an interesting physical picture of sediment transport along the cross-shore profile. According to the criterion in curly brackets, determined from quantities measured well outside the surf zone, sediment "is transported offshore or onshore along the full length of the active profile at a specific instant in time" (LK, p. 166). As noted by LK (p. 166), this formulation is less likely to correctly predict the actual direction of transport as the beach approaches equilibrium, where small departures from equilibrium may create large qualitative changes in sediment transport (i.e., change the direction of transport). Under conditions of relatively great profile disequilibrium, this deficiency in the model is thought to be less important.

The derivation of expressions for  $E$  and  $E_{eq}$  follows Dean (1977, Appendix III), assuming linear wave theory:

$$E = \frac{1}{h} \frac{d}{dx} \left[ \frac{1}{8} \rho g H_s^2 \sqrt{gh} \right] \quad (26)$$

where  $H_s$  is the mean significant wave height. A useful form for calculating  $E$  from field measurements is

$$E = \frac{\rho g^{\frac{3}{2}}}{8h^{\frac{3}{2}}} \frac{d(H_s^2 h)}{dx} \quad (27)$$

although discretization of Equation 27 using less than three relatively closely spaced measurement sites will yield predictions of transport at a point interpolated between actual measurement points.

The expression for  $E_{eq}$  is (LK, p. 168)

$$E_{eq} = \frac{5}{24} \rho g^{\frac{3}{2}} \gamma_b^2 A^{\frac{3}{2}} \quad (28)$$

where  $\gamma_b$  is the breaker index, the ratio between wave height and water depth at breaking. Equation 28 is derived by substituting  $\gamma_b h$  for  $H_s$  in Equation 26 and differentiating with respect to  $x$  to obtain

$$E_{eq} dx = \frac{5}{16} \rho g^{\frac{3}{2}} \gamma_b^2 h^{\frac{1}{2}} dh \quad (29)$$

which can be integrated to give Equation 23.

The profile shape parameter  $A$  is then

$$A = \frac{24}{5} \frac{E_{eq}^{\frac{2}{3}}}{\rho g^{\frac{2}{3}} \gamma^2} \quad (30)$$

which can in turn be solved for  $E_{eq}$ .

Dean (1987) proposed an empirical dimensional formula for determining  $A$  as a function of settling velocity  $W$

$$A = 0.067W^{0.44} \quad (31)$$

in which  $A$  has units of meters<sup>1/3</sup> when the settling velocity  $W$  is expressed in cm s<sup>-1</sup>. The fit to the data used by Dean (1987, p. 11) appears good for  $D_{50} > 1$  mm, and less good for smaller grains likely to be transported in suspension. It is not clear whether LK use Equation 31 to determine  $A$  or the earlier relationship proposed by Moore (1982).

### SBEACH parameters

Various parameters for SBEACH have been determined for laboratory and field application. The particular choice of wave height, for example, depends on the intended application. This section summarizes and discusses parameters and measurements relevant to field application of SBEACH.

The recommended value of the transport rate coefficient  $K$  appearing in Equation 24 was determined for both laboratory and field situations by minimizing the sum of squares of deviations between measured and predicted profile elevations. Seven erosional (bar-forming) profiles generated in large wave laboratory facilities were used to determine an optimal value for  $K$  of  $1.6 \cdot 10^{-6} \text{ m}^4 \text{ N}^{-1}$ ; in the field, a value of  $K=0.7 \cdot 10^{-6} \text{ m}^4 \text{ N}^{-1}$  minimized the discrepancy between measured and calculated profiles. Moore (1982) determined a value of  $K=2.2 \cdot 10^{-6} \text{ m}^4 \text{ N}^{-1}$  using laboratory and field data in a similar model (no explicit term proportional to  $dh/dx$ ). Intercomparison between SBEACH and the model of Kriebel (1986) produced an optimal  $K=8.7 \cdot 10^{-6} \text{ m}^4 \text{ N}^{-1}$  for the Kriebel model. Because it is not possible to relate the predicted transport rate to any physically measureable quantity, finding an optimal calibration procedure for  $K$  is difficult. Finally, the breaker index  $\gamma_b$  is set equal to unity for field application (LK, p. 207). Note that  $\gamma_b$  differs from  $\gamma$ , the ratio of wave height to water depth across the surf zone (Thornton and Guza 1983).

### Ackers and White (1973)

The semi-empirical sediment transport model of Ackers and White (1973) was developed for steady, uniform open-channel flows, and is considered here because it is widely used in a variety of applications. The model describes total sediment transport in terms of three dimensionless groups related to the size, mobility, and transport of grains having diameters greater than 0.04 mm. Nearly 1,000 flume experiments using well-sorted sediment in steady, uniform flows having depths up to 0.4 m form the empirical basis for the model. A single test of the model under a quasi-unidirectional flow regime was performed during one tidal cycle in an estuary (p. 2053):

... theoretical predictions appear to exaggerate the net transport in a landward direction. This is probably a result of the asymmetrical nature of the ebb/flood cycle and there is clearly a need for further investigation when the flow is time-dependent.

Use of the Ackers and White formulation is thus not recommended for nearshore oscillatory flows. The following sections outline the procedure for computing sediment transport of fine grains in suspension. A dimensionless grain diameter  $D_*$ , mobility parameter  $F_*$ , and transport parameter  $G_*$  are calculated as follows:

The dimensionless grain diameter  $D_*$  is

$$D_* = D \left[ \frac{g(\rho_s - \rho)}{\rho \nu^2} \right]^{1/3} \quad (32)$$

where  $D$  is the medium grain size and  $\nu$  is the kinematic viscosity. Ancillary parameters  $n$ ,  $A_g$ ,  $C_g$ , and  $m$  are empirically derived functions of  $D_*$ :

$$n = 1 - 0.56 \log D_* \quad (33)$$

$$A_g = 0.23 D_*^{-1/2} + 0.14 \quad (34)$$

$$\log C_g = 2.86 \log D_* - (\log D_*)^2 - 3.53 \quad (35)$$

$$m = 9.66 D_*^{-1} + 1.34 \quad (36)$$

Taking  $n \approx 1$  for fine suspended sediments, the mobility parameter  $F_*$  is simply the square root of the Shield's parameter  $\theta$

$$F_* = \frac{u_*}{[gD(\rho_s - \rho)]^{1/2}} = \theta^{1/2} \quad (37)$$

and the transport parameter  $G_*$  is then

$$G_* = C_s \left[ \frac{F_*}{A_g} - 1 \right]^m \quad (38)$$

Ackers and White formulate the sediment flux in terms of the sediment concentration in parts per million by weight of fluid flux, and thus the immersed sediment transport rate is

$$\langle i \rangle = 10^{-6} \frac{\rho_s - \rho}{\rho_s} C_s \left( \frac{F_*}{A_g} - 1 \right)^m \left( \frac{\rho_s D \bar{u}}{\rho h u_*} \right) (\rho \bar{u} h) \quad (39)$$

where  $\bar{u}$  is the depth-averaged velocity, and the various factors of  $h$ ,  $\rho$  and  $\rho_s$  have been left unsimplified for clarity. For field application, the shear velocity  $u_*$  in Equation 39 might be computed using Equation 4 and a suitable choice for  $c_f$ .



**Table 1**  
**Transport Formulae**

Bowen (1980)<sup>1</sup> 
$$\langle \vec{\tau} \rangle = \rho c_f \frac{e_s}{W} \left[ \langle |\vec{u}|^3 \vec{u} \rangle + \frac{\beta}{W} \langle |\vec{u}|^5 \rangle \hat{i} \right]$$

Bailard (1981)<sup>2</sup> 
$$\langle \vec{\tau} \rangle = \rho c_f \frac{e_s}{W} \left[ \langle |\vec{u}|^3 \vec{u} \rangle + \frac{e_s \tan \beta}{W} \langle |\vec{u}|^5 \rangle \hat{i} \right]$$

Roelvink and Stive (1989) (cross-shore transport only)

$$\langle i \rangle = \rho c_f \frac{e_s}{W} \left[ \langle |\vec{u}|^3 \vec{u} \rangle + K_1 \right]$$

where 
$$K_1 = \beta_d \{ k_i [ \exp (h/H_{rms}) - 1 ]^{-1} \}^{\frac{3}{2}}$$

Larson and Kraus (1989) (cross-shore transport only)

$$\langle i \rangle = \text{sgn} \left\{ M \left[ \frac{\langle H_0 \rangle}{WT} \right]^3 - \frac{2\pi \langle H_0 \rangle}{gT^2} \right\} (\rho_s - \rho) gK \left[ E - E_{eq} + \frac{\epsilon_\beta}{K} \frac{dh}{dx} \right]$$

when  $E > E_{eq} - \frac{\epsilon_\beta}{K} \frac{dh}{dx}$ , and zero otherwise.

Ackers and White (1973)

$$\langle i \rangle = 10^{-6} \frac{\rho_s - \rho}{\rho_s} C_s \left( \frac{F_*}{A_g} - 1 \right)^m \left( \frac{\rho_s D \bar{u}}{\rho h u_*} \right) (\rho \bar{u} h)$$

<sup>1</sup> Bagnold (1963) is functionally equivalent, but not elaborated in the same detail.

<sup>2</sup> Humiston (1993) follows Bailard (1981).

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# Appendix A

## Notation

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The notation adopted herein attempts to use commonly accepted symbols whenever possible. Coordinate systems are all right-handed, but in some cases the  $x$ -coordinate is parallel to the sloping planar bed while in others the  $x$ - $y$  coordinate plane is horizontal. Except where noted, angle brackets are used to denote a temporal or ensemble average; overbars indicate a spatial average; tildes indicate an oscillatory component; and primes indicate a fluctuating component. The subscripts  $b$  and  $0$  refer to quantities evaluated at the bed, although the location of the nominal bed elevation differs in various theories. The symbol  $[\emptyset]$  denotes a dimensionless quantity.

$c$	Wave phase velocity	$[LT^{-1}]$
$c_f$	Friction factor or bed drag coefficient	$[\emptyset]$
$c_g$	Group velocity	$[LT^{-1}]$
$c_r$	Coefficient in Equation 21	$[\emptyset]$
$dh/dx$	Bed slope	$[\emptyset]$
$e_b$	Bed-load efficiency factor	$[\emptyset]$
$e_s$	Suspended load efficiency factor	$[\emptyset]$
$f$	Wave frequency	$[T^{-1}]$
$f_p$	Peak wave frequency	$[T^{-1}]$
$f_w$	Wave friction factor	$[\emptyset]$
$g$	Gravitational acceleration	$[L T^{-2}]$
$h$	Water depth	$[L]$
$i$	Immersed weight sediment transport rate per unit width	$[ML^{-1} T^{-1}]$

$i_\theta$	Immersed weight sediment transport rate in $\theta$ direction	$[ML^{-1} T^{-1}]$
$\hat{i}$	Unit vector in $x$ -direction (offshore)	$[\emptyset]$
$k$	Wave number ( $= 2 \pi L^{-1}$ )	$[L^{-1}]$
$k_t$	Turbulent kinetic energy per unit mass	$[L^2 T^{-2}]$
$m$	Exponent defined in Equation 36	$[\emptyset]$
$n$	Exponent defined in Equation 33	$[\emptyset]$
$s$	Ratio of grain and fluid densities	$[\emptyset]$
$t$	Time	$[T]$
$u$	Water velocity in $x$ -direction	$[LT^{-1}]$
$u_*$	Friction or shear velocity	$[LT^{-1}]$
$u_m$	Maximum orbital wave velocity	$[LT^{-1}]$
$u_\theta$	Current velocity in $\theta$ -direction	$[LT^{-1}]$
$\vec{u}$	Instantaneous fluid velocity	$[LT^{-1}]$
$v$	Water velocity in $y$ -direction	$[LT^{-1}]$
$w$	Water velocity in $z$ -direction	$[LT^{-1}]$
$x$	Horizontal coordinate (+offshore)	$[L]$
$y$	Horizontal coordinate	$[L]$
$z$	Vertical coordinate (+up)	$[L]$
$A$	Profile shape parameter	$[L^{1/3}]$
$A_g$	Constant defined in Equation 34	$[\emptyset]$
$B$	Coefficient in Equation 13	$[\emptyset]$
$C_g$	Constant defined in Equation 35	$[\emptyset]$
$D$	Mean sediment grain diameter	$[L]$
$D_*$	Dimensionless sediment grain diameter	$[\emptyset]$
$E$	Energy dissipation rate per unit volume	$[M L^{-1} T^{-3}]$

$E_{eq}$	Equilibrium energy dissipation rate per unit volume	$[M L^{-1} T^{-3}]$
$F_*$	Dimensionless mobility parameter	$[\emptyset]$
$G_*$	Dimensionless transport parameter	$[\emptyset]$
$H$	Wave height	$[L]$
$H_0$	Deepwater wave height	$[L]$
$H_s$	Significant wave height	$[L]$
$H_{rms}$	Root-mean-square wave height	$[L]$
$L$	Wave length	$[L]$
$K$	Coefficient in Equation 24	$[L^3 M^{-1} T^2]$
$K_B$	Coefficient in Equation 3	$[L^{-1} T^2]$
$K_1$	Coefficient in Equation 6	$[L^3 T^{-2}]$
$K_2$	Coefficient in Equation 6	$[L^{-1} T]$
$M$	Coefficient in Equation 24	$[\emptyset]$
$N$	Volume concentration	$[\emptyset]$
$N_{bed}$	Volume concentration of randomly packed particles	$[\emptyset]$
$S$	Energy slope in unidirectional flow	$[\emptyset]$
$R$	Rouse number	$[\emptyset]$
$T$	Wave period	$[T]$
$U_1$	Perturbation velocity	$[LT^{-1}]$
$W$	Sediment fall velocity	$[LT^{-1}]$
$\alpha$	Wave angle (deviation from normal)	$[\text{radians}]$
$\beta$	Bed slope	$[\emptyset]$
$\beta_d$	Coefficient in Equation 11	$[\emptyset]$
$\epsilon_d$	Energy dissipation rate	$[M T^{-3}]$
$\epsilon\beta$	Coefficient in Equation 24	$[M T^{-3}]$

$\gamma$	Ratio of wave height to water depth	[Ø]
$\gamma_b$	Ratio of wave height to water depth at breaking	[Ø]
$\kappa$	von Karman's constant (= 0.4)	[Ø]
$\theta$	Shield's parameter	[Ø]
$\rho$	Fluid density	[M L <sup>-3</sup> ]
$\rho_s$	Sediment density	[M L <sup>-3</sup> ]
$\sigma$	Radial wave frequency	[T <sup>-1</sup> ]
$\tau_b$	Bed shear stress	[M L <sup>-1</sup> T <sup>-2</sup> ]
$\nu$	Kinematic viscosity	[L <sup>2</sup> T <sup>-1</sup> ]
$\nu_s$	Sediment diffusivity	[L <sup>2</sup> T <sup>-1</sup> ]
$\omega$	Wave or stream power	[M T <sup>-3</sup> ]

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